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Application of p-adic analysis to models of breaking of replica symmetry

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Abstract. Methods of p-adic analysis are applied to the investigation of spontaneous symmetry breaking in the models of spin glasses. A p-adic expression for the Parisi replica matrix is given and, moreover, the Parisi replica matrix in models of spontaneous breaking of the replica symmetry in the simplest case is expressed in the form of the Vladimirov operator of p-adic fractional differentiation. Also, the model of hierarchical diffusion (that was proposed to describe relaxation of spin glasses) is investigated using p-adic analysis.

1. Introduction

Numerous works, for example [1–3], discuss the application of ultrametrics to the investigation of spin glasses. The most important example of ultrametric space is the field of p-adic numbers, for an introduction to p-adic analysis, see [4]. p-adic mathematical physics attracts a great deal of interest, see [4–6]. For instance, p-adic models in string theory were introduced, see [7, 8], and p-adic quantum mechanics [9] and p-adic quantum gravity [10] were investigated. In the present paper we apply the methods of p-adic analysis to investigate the spontaneous symmetry breaking in the models of spin glasses. We obtain the following results:

(1) A p-adic expression for the replica matrix Q_{ab} is found. It has the form $Q_{ab} = q_k$, where $k = \log_p |l(a) - l(b)|_p$: the notation is expressed below. It is shown that the replica matrix in the Parisi form [1] in the models of spontaneous breaking of the replica symmetry in the simplest case has the form of the Vladimirov operator of p-adic fractional differentiation [4].

(2) The model of hierarchical diffusion that was used in [11] to describe relaxation of spin glasses in our approach takes the form of the model of p-adic diffusion. For instance, we reproduce the results of [11] using the methods of p-adic analysis.

The results of the present paper were partially presented in [12]. After completion of this work we received a paper by G Parisi and N Sourlas [13] where similar results are derived (see the discussion in section 2).

The models of spontaneous breaking of the replica symmetry are used for the investigation of spin glasses [1–3]. The breaking of symmetry in such models is described by the replica $n \times n$ matrix $\boldsymbol{Q} = (Q_{ab})$ in the Parisi form [1]. This matrix appears as follows. Let us consider the set of integer numbers m_i, $i = 1, \ldots, N$, where m_i/m_{i-1} are integers for $i > 1$ and n/m_i are also integers. The matrix element of the replica matrix [1] is defined as follows:

$$Q_{aa} = 0 \qquad Q_{ab} = q_i \qquad \left[\frac{a}{m_i}\right] \neq \left[\frac{b}{m_i}\right] \qquad \left[\frac{a}{m_{i+1}}\right] = \left[\frac{b}{m_{i+1}}\right]. \tag{1}$$

Here $[\cdot]$ is the function of integer part (we understand the integer part $[x]$ to be as follows: $[x]-1 \leqslant x \leqslant [x]$ where $[x]$ is the integer) and q_i are some (real) parameters. An example of a matrix of this kind for $m_i/m_{i-1} = 2$ and $n = 2^N$ has the form

$$Q = \begin{pmatrix} 0 & q_1 & q_2 & q_2 & q_3 & q_3 & q_3 & q_3 & \cdots \\ q_1 & 0 & q_2 & q_2 & q_3 & q_3 & q_3 & q_3 & \cdots \\ q_2 & q_2 & 0 & q_1 & q_3 & q_3 & q_3 & q_3 & \cdots \\ q_2 & q_2 & q_1 & 0 & q_3 & q_3 & q_3 & q_3 & \cdots \\ q_3 & q_3 & q_3 & q_3 & 0 & q_1 & q_2 & q_2 & \cdots \\ q_3 & q_3 & q_3 & q_3 & q_1 & 0 & q_2 & q_2 & \cdots \\ q_3 & q_3 & q_3 & q_3 & q_2 & q_2 & 0 & q_1 & \cdots \\ q_3 & q_3 & q_3 & q_3 & q_2 & q_2 & q_1 & 0 & \cdots \\ \cdots \end{pmatrix}. \tag{2}$$

In the present paper we discuss the replica matrix (2) (more precisely, the generalization of this example for the case of $p^N \times p^N$ matrices) using the language of p-adic analysis. This allows one to give the natural interpretation for (2) as the operator that can be diagonalized by the p-adic Fourier transform. In particular, this gives the spectrum of matrix (2). In the limit of infinite breaking of the replica symmetry $N \to \infty$, the dimension p^N of the replica matrix tends to infinity, but the p-adic norm of the dimension $|p^N|_p = p^{-N}$ tends to zero. The conjecture made by Volovich [14] is that this phenomenon might explain the paradoxical fact that in the replica method the dimension of the replica matrix in the limit of infinite breaking of the replica symmetry tends to zero.

Here we give a brief review of p-adic analysis. The field Q_p of p-adic numbers is the completion of the field of rational numbers Q with respect to the p-adic norm on Q. This norm is defined in the following way. An arbitrary rational number x can be written in the form $x = p^\gamma \frac{m}{n}$ where m and n are not divisible by p. The p-adic norm of the rational number $x = p^\gamma \frac{m}{n}$ is equal to $|x|_p = p^{-\gamma}$.

The most interesting property of the field of p-adic numbers is ultrametricity. This means that Q_p obeys the strong triangle inequality

$$|x+y|_p \leqslant \max(|x|_p, |y|_p).$$

We consider discs in Q_p of the form $\{x \in Q_p : |x - x_0|_p \leqslant p^{-k}\}$. For example, the ring Z_p of integer p-adic numbers is the disc $\{x \in Q_p : |x|_p \leqslant 1\}$, which is the completion of integers with the p-adic norm. The main properties of discs in arbitrary ultrametric space are as follows:

(1) Every point of a disc is the centre of this disc.
(2) Two discs either do not intersect or one of these discs contains the other.

The p-adic Fourier transform F of the function $f(x)$ is defined as follows:

$$F[f](\xi) = \tilde{f}(\xi) = \int_{Q_p} \chi(\xi x) f(x)\, \mathrm{d}\mu(x)$$

where $\mathrm{d}\mu(x)$ is the Haar measure. The inverse Fourier transform has the form

$$F^{-1}[\tilde{g}](x) = \int_{Q_p} \chi(-\xi x)\tilde{g}(\xi)\, \mathrm{d}\mu(\xi).$$

Here $\chi(\xi x) = \exp(\mathrm{i}\xi x)$ is the character of the field of p-adic numbers. For example, the Fourier transform of the indicator function $\Omega(x)$ of the disc of radius one with its centre at zero (this is a function that equals one on the disc and zero outside the disc) is the function of the same type:

$$\tilde{\Omega}(\xi) = \Omega(\xi).$$

In the present paper we use the following Vladimirov operator D_x^α of the fractional p-adic differentiation, which is defined [4] as

$$D_x^\alpha f(x) = F^{-1} \circ |\xi|_p^\alpha \circ F[f](x) = \frac{p^\alpha - 1}{1 - p^{-1-\alpha}} \int_{Q_p} \frac{f(x) - f(y)}{|x-y|_p^{1+\alpha}} \, \mathrm{d}\mu\,(y). \tag{3}$$

Here F is the (p-adic) Fourier transform, the second equality holds for $\alpha > 0$.

For further reading on the subject of p-adic analysis, see [4].

2. The replica matrix

Let us describe the model of the replica symmetry breaking using the language of p-adic analysis. We show that the replica matrix $\boldsymbol{Q} = (Q_{ab})$ can be considered as an operator in the space of functions on the finite set consisting of p^N points with the structure of the ring $p^{-N}Z/Z$. The ring $p^{-N}Z/Z$ can be described as a set with the elements

$$x = \sum_{j=1}^{N} x_j p^{-j} \qquad 0 \leqslant x_j \leqslant p-1$$

with natural operations of addition and multiplication up to modulus 1. Let us consider the p-adic norm on this ring (the distance can take values 0, $p, \ldots, p^N$). We consider the following construction and introduce one-to-one correspondence:

$$l : 1, \ldots, p^N \to p^{-N}Z/Z$$

$$l^{-1} : \sum_{j=1}^{N} x_j p^{-j} \mapsto 1 + p^{-1} \sum_{j=1}^{N} x_j p^j \qquad 0 \leqslant x_j \leqslant p-1.$$

Formula (1) takes the form

$$Q_{aa} = 0 \qquad Q_{ab} = q_i \qquad \left[\frac{a}{p^{i-1}}\right] \neq \left[\frac{b}{p^{i-1}}\right] \qquad \left[\frac{a}{p^i}\right] = \left[\frac{b}{p^i}\right]. \tag{4}$$

Let us prove the following theorem.

Theorem. *The matrix element Q_{ab} defined by (4) depends only on the p-adic distance between $l(a)$ and $l(b)$:*

$$Q_{ab} = \rho(|l(a) - l(b)|_p)$$

where $\rho(p^k) = q_k$, $\rho(0) = 0$.

Proof. The condition $[\frac{a}{p^i}] = [\frac{b}{p^i}]$ expressed in our notation has the form

$$\left[\frac{1 + p^{-1}\sum_{j=1}^{N} a_j p^j}{p^i}\right] = \left[\frac{1 + p^{-1}\sum_{j=1}^{N} b_j p^j}{p^i}\right].$$

This means that $a_j = b_j$ for $j > i$. The condition $[\frac{a}{p^{i-1}}] \neq [\frac{b}{p^{i-1}}]$ means that $a_i \neq b_i$. However, these two conditions both mean that $|l(a) - l(b)|_p = p^{-i}$. The matrix element of the replica matrix Q_{ab} depends only on the p-adic distance $|l(a) - l(b)|_p$: if $|l(a) - l(b)|_p$ equals p^{-k} then $Q_{ab} = q_k$ and the statement of the theorem follows.

The replica matrix (Q_{ab}) acts on functions on $p^{-N}Z/Z$ as on vectors with matrix elements f_b where $b = l(y)$, $b = 1, \ldots, p^N$. The action of the replica matrix in the space of functions on $p^{-N}Z/Z$ takes the form

$$\boldsymbol{Q} f(x) = \int_{p^{-N}Z/Z} \rho(|x-y|_p) f(y) \, \mathrm{d}\mu\,(y) \tag{5}$$

where the measure $\mathrm{d}\mu\,(y)$ of one point equals one and $f_b = f(l(b))$ (because we can consider the index b of the vector as the index of the first column of the matrix (Q_{ab})).

It is easy to see that operators which take the form of (5) have the following properties:

(1) The operators (5) commute with shift operators. This means that the operators (5) can be diagonalized by the Fourier transform (in our case this is the discrete Fourier transform).
(2) The function ρ depends on the p-adic norm of the argument.
(3) $\rho(0) = 0$.

The language of p-adic analysis allows us to describe the natural generalization of operator (5). This generalization has the operator form

$$\boldsymbol{Q} f(x) = \int_{Q_p} \rho(|x-y|_p) f(y)\,\mathrm{d}\mu\,(y) \tag{6}$$

where the function ρ obeys properties (1) and (2) (an analogue of property (3) is considered later). Here, and in what follows, we stipulate the agreement that we use the same notation (without special comments) for analogous values in the discrete and the continuous (p-adic) cases.

In [13] Parisi and Sourlas discuss an analogous construction of the p-adic generalization of the Parisi matrix connecting it with the famous replica approach limit $n \to 0$, where n is the dimension of replica matrix. The summation of elements in the line of the Parisi matrix was performed to get

$$\sum_b Q_{ab} = \sum_{i=1}^{N} (p-1) p^{i-1} q_i .$$

One rewrites this as

$$\sum_{i=-\infty}^{N} (p-1) p^{i-1} q_i - \sum_{i=-\infty}^{0} (p-1) p^{i-1} q_i \tag{7}$$

and takes the formal limit $N \to -\infty$ (that corresponds to the limit $n \to 0$ for the dimension of replica matrix $n = p^N$). The first term in (7) disappears in the limit and the second term exactly gives the p-adic integral [13] $-p\int_{|x|_p \leqslant 1} \rho(|x|_p)\,\mathrm{d}\mu\,(x)$, where $\rho(p^k) = q_k$, $k \leqslant 0$.

It is easy to see that the character $\chi(kx)$ is the generalized eigenvector for the operator (6), if $\rho(|x|_p) \in L^1(Q_p)$. Thus, the operator (6) can be diagonalized by the p-adic Fourier transform F: $\boldsymbol{Q} f(x) = F^{-1} \circ \gamma(\xi) \circ F[f](x)$. From property (2) it follows that the function γ depends only on the p-adic norm of the argument: $\gamma = \gamma(|\xi|_p)$. Therefore, we get

$$\boldsymbol{Q} f(x) = F^{-1} \circ \gamma(|\xi|_p) \circ F[f](x).$$

□

3. The model of hierarchical diffusion

We now reproduce (partially) the results of [11] using the methods of p-adic analysis. In [11] the relaxation of spin glasses was described using the following model of hierarchical diffusion. Let us consider 2^N points (we also consider the more general case of p^N points, where $p > 0$ is prime), separated by energy barriers. The energy barriers have the following form. Let us enumerate the points by integer numbers starting from 0 to $2^N - 1$ (analogously, from 0 to $p^N - 1$). Let us consider the increasing sequence of energy barriers (non-negative numbers) $0 = \Delta_0 < \Delta_1 < \Delta_2 < \cdots < \Delta_k < \cdots$. We define the energy barriers on the set of p^N points according to the following rule: if $a - b$ is divisible by p^k then the barrier between the ath and bth points is equal to Δ_k.

The hierarchical diffusion is described by the ensemble of particles that jump over the above-described set of p^N points. Let us define the probability q_i of transition (or jump) over the energy barrier Δ_i in the following way: $q_i = \exp(-\Delta_i)$, $i = 1, 2, \ldots$. Then, the transition probability matrix will be equal (up to additive constant) to matrix $\boldsymbol{Q}$ of the form (2).

We denote the density of particles at the ath point as $f_a(t)$ and the vector with elements that are equal to the densities at all points as $\boldsymbol{f}(t)$. We define the dynamics of the model using the following differential equation [11]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{f}(t) = (\boldsymbol{Q} - \lambda_0 \boldsymbol{I})\boldsymbol{f}(t) \tag{8}$$

where the $N \times N$ matrix $\boldsymbol{Q}$ for $p = 2$ has the form (2) of the replica matrix for the model of the replica symmetry breaking, $\boldsymbol{I}$ is the unity matrix, and λ_0 is the eigenvalue of the matrix $\boldsymbol{Q}$ that corresponds to the eigenvector with equal matrix elements. This choice of transition probability matrix is defined by the law of particle number conservation (that is an analogue of property (3)).

Application of the technique developed in section 2 allows us to write equation (8) in the form

$$\frac{\mathrm{d}}{\mathrm{d}t} f(x,t) = \int_{p^{-N}Z/Z} (f(y,t) - f(x,t))\rho(|x-y|_p)\,\mathrm{d}\mu\,(y) \tag{9}$$

where $f_a(t) = f(l(a), t)$. For example, for the above-considered $q_i = \exp(-\Delta_i)$, $i = 1, 2, \ldots$ and for the linear dependence of the barrier energy $\Delta_i = \mathrm{i}(1+\alpha)\ln p$ on i we get $\rho(|x|_p) = |x|_p^{-1-\alpha}$ and equation (9) takes the form

$$\frac{\mathrm{d}}{\mathrm{d}t} f(x,t) = \int_{p^{-N}Z/Z} \frac{f(y,t) - f(x,t)}{|x-y|_p^{1+\alpha}}\,\mathrm{d}\mu\,(y). \tag{10}$$

On the right-hand side (RHS) of equation (10) we get the discretization of the Vladimirov operator D_x^α (3), of the fractional p-adic differentiation, see [4].

In [11] the Cauchy problem for the equation (10) with the initial condition $f(x,0) = \delta_{x0}$ was investigated. The time dependence of the $P_0(t)$ value that in the p-adic notation has the form

$$P_0(t) = f(0,t) = \int_{p^{-N}Z/Z} \delta_{y0} f(y,t)\,\mathrm{d}\mu\,(y)$$

was found. In the present paper we calculate the $P_0(t)$ value using the method of p-adic analysis.

The p-adic generalization of equation (9) has the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t} f(x,t) = \int_{Q_p} (f(y,t) - f(x,t))\rho(|x-y|_p)\,\mathrm{d}\mu\,(y). \tag{11}$$

Let us describe how to get the spectrum of operator $\boldsymbol{D}$ in the RHS of (11) (or the spectrum of relaxation times for the model of hierarchical diffusion [11], describing spin glasses). We use the p-adic Fourier transform. It is easy to see that the character $\chi(kx)$ is the generalized eigenfunction for the operator in the RHS of (11) if $\rho(|x|_p) \in L^1(Q_p \backslash U_\epsilon)$, where U_ϵ is the arbitrary neighbourhood of zero, or equivalently if $\forall k \in Z$ the series $\sum_{i=k}^{\infty} |\rho(p^i)| p^{-i}$ converges. For instance, 1 is the eigenfunction for the eigenvalue that equals zero. The proof is as follows:

$$\boldsymbol{D}\chi(kx) = \int_{Q_p} (\chi(ky) - \chi(kx))\rho(|x-y|_p)\,\mathrm{d}\mu\,(y)$$

$$= \chi(kx)\int_{Q_p}(\chi(k(y-x))-1)\rho(|x-y|_p)\,\mathrm{d}\mu\,(y)$$
$$= \chi(kx)\int_{Q_p}(\chi(ky)-1)\rho(|y|_p)\,\mathrm{d}\mu\,(y).$$

To finish the proof we note that $\chi(ky)$ is a locally constant function that equals one in some neighbourhood of zero. Using the fact that the integral $\int_{|y|_p\leqslant p^i}\chi(ky)\,\mathrm{d}\mu\,(y) = p^i$, if $|k|_p \leqslant p^{-i}$ and equals zero if $|k|_p > p^{-i}$, we get

$$\boldsymbol{D}\chi(kx) = \left(-(1-p^{-1})\sum_{p^i\leqslant|k|_p} p^{-i}\rho(p^i) - \frac{p^{-1}}{|k|_p}\rho(|k|_p)\right)\chi(kx). \tag{12}$$

This relation shows the correspondence between the spectrum of relaxation times and the elements of the replica matrix in the form (2) (here $q_i = \rho(p^i)$). Relation (12) reproduces the result obtained in [11], where a more complicated technique was used.

Let us describe how to get the operator in the RHS of equation (9) using the analogous operator (in the RHS of equation (11)) on Q_p. Consider the finite-dimensional subspace $V_N \subset L^2(Q_p)$ of the following form. The subspace V_N consists of functions with zero average with support in $p^{-N}Z_p$ that are constants on discs of radius one. Therefore, the dimension of the subspace V_N equals $p^N - 1$. The operator at (11) maps this space into itself. At the subspace V_N, the operator in the RHS of equation (11) takes the form

$$\boldsymbol{D}f(x) = \int_{p^{-N}Z/Z}(f(y)-f(x))\rho(|x-y|_p)\,\mathrm{d}\mu\,(y)$$

which looks exactly like the operator in the RHS of equation (9). However, using this method we will not obtain equation (9) because the operator in the RHS of (9) acts in the dimension space that is larger, by one, than V_N. This space can be obtained from the space V_N by adding to V_N the function that equals one on the ball $p^{-N}Z_p$ (and zero outside).

Thus, the model presented in [11] (up to the comments made above) corresponds to the action of the operator of p-adic fractional differentiation at the subspace.

We investigate the following p-adic generalization of the model given in [11]. Let us consider the Cauchy problem for the p-adic generalization of equation (10):

$$\frac{\mathrm{d}}{\mathrm{d}t}f(x,t) + AD_x^{\alpha}f(x,t) = 0 \tag{13}$$

that has the form of the equation of p-adic diffusion that was investigated in [4]. We take the initial equation for (13) of the form

$$f(x,0) = \delta(x). \tag{14}$$

This means that we investigate the fundamental solution of equation (13). Fourier transform equation (13) then takes the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{f}(\xi,t) + A|\xi|_p^{\alpha}\tilde{f}(\xi,t) = 0.$$

The solution of this equation is $\tilde{f}(\xi,0)\mathrm{e}^{-A|\xi|_p^{\alpha}t}$. Because the Fourier transform of the δ-function with support in zero equals one, we finally get

$$\begin{aligned}\tilde{f}(\xi,t) &= \mathrm{e}^{-A|\xi|_p^{\alpha}t}\\ f(x,t) &= \int_{Q_p}\chi(-\xi x)\mathrm{e}^{-A|\xi|_p^{\alpha}t}\,\mathrm{d}\mu\,(\xi).\end{aligned} \tag{15}$$

As the p-adic generalization of $P_0(t)$ we consider the value

$$P_0(t) = \int_{|x|_p \leqslant 1} f(x,t)\,\mathrm{d}\mu(x) = \int_{Q_p} \Omega(x) f(x,t)\,\mathrm{d}\mu(x)$$

(we use the same notation) that for solution (15) takes the form

$$\int_{Q_p} \Omega(\xi)\mathrm{e}^{-A|\xi|_p^\alpha t}\,\mathrm{d}\mu(\xi) = \int_{|\xi|_p \leqslant 1} \mathrm{e}^{-A|\xi|_p^\alpha t}\,\mathrm{d}\mu(\xi) = (1-p^{-1})\sum_{k=0}^{\infty} p^{-k}\mathrm{e}^{-Ap^{-\alpha k}t}. \tag{16}$$

Our answer coincides with the answer obtained in [11] for (9). The value found in [11] (they use $p = 2$) has the form

$$P_0(t) = \lim_{n\to\infty}\left(2^{-n} + \frac{1}{2}\exp\left(\frac{R^{n+1}t}{1-R}\right)\sum_{m=0}^{n-1}\exp\left(-m\ln 2 - \frac{2-R}{1-R}R^{m+1}t\right)\right) \tag{17}$$

where $0 < R < 1$ is some constant. It is easy to see that (16) and (17) coincide for $R = p^{-\alpha}$ and $A = \frac{2-R}{1-R}$. We see that p-adic analysis allows us to investigate the models of hierarchical diffusion using the simple and natural formalism.

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